PHYSICS FORMULAS
2426

Electron = -1.602 19 × 10^{-19} C = 9.11 × 10^{-31} kg
Proton = 1.602 19 × 10^{-19} C = 1.67 × 10^{-27} kg
Neutron = 0 C = 1.67 × 10^{-27} kg
6.022 × 10^{23} atoms in one atomic mass unit
e is the elementary charge: 1.602 19 × 10^{-19} C
Potential Energy, velocity of electron: $\mathrm{PE} = eV = ½mv^2$

1V = 1J/C   1N/C = 1V/m   1J = 1 N·m = 1 C·V
1 amp = 6.21 × 10^{18} electrons/second = 1 Coulomb/second
1 hp = 0.756 kW   1 N = 1 T·A·m   1 Pa = 1 N/m^2
Power = Joules/second = $I^2R = IV$ [watts *W*]

Quadratic Equation: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Kinetic Energy [J]: $KE = \frac{1}{2}mv^2$

[Natural Log: when $e^b = x$, $\ln x = b$]
m: 10^{-3} µ: 10^{-6} n: 10^{-9} p: 10^{-12} f: 10^{-15} a: 10^{-18}

Addition of Multiple Vectors:

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$ Resultant = Sum of the vectors

$\vec{R}_x = \vec{A}_x + \vec{B}_x + \vec{C}_x$ *x*-component $A_x = A \cos\theta$

$\vec{R}_y = \vec{A}_y + \vec{B}_y + \vec{C}_y$ *y*-component $A_y = A \sin\theta$

$R = \sqrt{R_x{}^2 + R_y{}^2}$ Magnitude (length) of *R*

$q_R = \tan^{-1}\dfrac{R_y}{R_x}$ *or* $\tan q_R = \dfrac{R_y}{R_x}$ Angle of the resultant

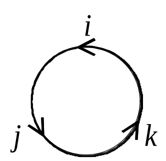
Multiplication of Vectors:

Cross Product or Vector Product:

$i \times j = k$ $j \times i = -k$

$i \times i = 0$

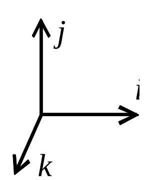
Positive direction:



Dot Product or Scalar Product:

$i \cdot j = 0$ $i \cdot i = 1$

$\mathbf{a} \cdot \mathbf{b} = ab\cos q$



Derivative of Vectors:

Velocity is the derivative of position with respect to time:

$$\mathbf{v} = \frac{d}{dt}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

Acceleration is the derivative of velocity with respect to time:

$$\mathbf{a} = \frac{d}{dt}(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}) = \frac{dv_x}{dt}\mathbf{i} + \frac{dv_y}{dt}\mathbf{j} + \frac{dv_z}{dt}\mathbf{k}$$

Rectangular Notation: $Z = R \pm jX$ where $+j$ represents inductive reactance and $-j$ represents capacitive reactance. For example, $Z = 8 + j6\Omega$ means that a resistor of 8Ω is in series with an inductive reactance of 6Ω.

Polar Notation: $Z = M \angle q$, where M is the magnitude of the reactance and q is the direction with respect to the horizontal (pure resistance) axis. For example, a resistor of 4Ω in series with a capacitor with a reactance of 3Ω would be expressed as 5 ∠-36.9° Ω.

In the descriptions above, impedance is used as an example. Rectangular and Polar Notation can also be used to express amperage, voltage, and power.

To convert from **rectangular to polar** notation:

Given: X - jY (careful with the sign before the "j")

Magnitude: $\sqrt{X^2 + Y^2} = M$

Angle: $\tan\theta = \dfrac{-Y}{X}$ (negative sign carried over from rectangular notation in this example)

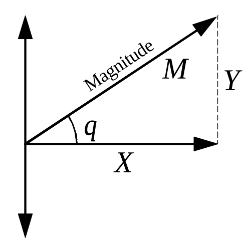
Note: Due to the way the calculator works, if X is negative, you must **add 180°** after taking the inverse tangent. If the result is greater than 180°, you may optionally subtract 360° to obtain the value closest to the reference angle.

To convert from **polar to rectangular** (j) notation:

Given: $M \angle q$

X Value: $M\cos\theta$

Y (j) Value: $M\sin\theta$



In conversions, the j value will have the same sign as the q value for angles having a magnitude < 180°.

Use rectangular notation when adding and subtracting.

Use polar notation for multiplication and division. Multiply in polar notation by multiplying the magnitudes and adding the angles. Divide in polar notation by dividing the magnitudes and subtracting the denominator angle from the numerator angle.

ELECTRIC CHARGES AND FIELDS

Coulomb's Law: [Newtons *N*]

$$F = k\frac{|q_1||q_2|}{r^2}$$

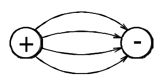
where:
- F = force on one charge by the other[*N*]
- $k = 8.99 \times 10^9$ [$N{\cdot}m^2/C^2$]
- q_1 = charge [*C*]
- q_2 = charge [*C*]
- r = distance [*m*]

Electric Field: [Newtons/Coulomb or Volts/Meter]

$$E = k\frac{|q|}{r^2} = \frac{|F|}{|q|}$$

where:
- E = electric field [*N/C* or *V/m*]
- $k = 8.99 \times 10^9$ [$N{\cdot}m^2/C^2$]
- q = charge [*C*]
- r = distance [*m*]
- F = force

Electric field lines radiate outward from positive charges. The electric field is zero inside a conductor.



Relationship of k to $\in_0$:

$$k = \frac{1}{4p \in_0}$$

where:
- $k = 8.99 \times 10^9$ [$N{\cdot}m^2/C^2$]
- $\in_0$ = permittivity of free space 8.85×10^{-12} [$C^2/N{\cdot}m^2$]

Electric Field due to an Infinite Line of Charge: [*N/C*]

$$E = \frac{l}{2p \in_0 r} = \frac{2kl}{r}$$

- E = electric field [*N/C*]
- l = charge per unit length [*C/m*}
- $\in_0$ = permittivity of free space 8.85×10^{-12} [$C^2/N{\cdot}m^2$]
- r = distance [*m*]
- $k = 8.99 \times 10^9$ [$N{\cdot}m^2/C^2$]

Electric Field due to ring of Charge: [*N/C*]

$$E = \frac{kqz}{(z^2 + R^2)^{3/2}}$$

or if z >> R, $E = \dfrac{kq}{z^2}$

- E = electric field [*N/C*]
- $k = 8.99 \times 10^9$ [$N{\cdot}m^2/C^2$]
- q = charge [*C*]
- z = distance to the charge [*m*]
- R = radius of the ring [*m*]

Electric Field due to a disk Charge: [*N/C*]

$$E = \frac{s}{2\in_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

- E = electric field [*N/C*]
- s = charge per unit area [*C/m*^{2}}
- $\in_0 = 8.85 \times 10^{-12}$ [$C^2/N{\cdot}m^2$]
- z = distance to charge [*m*]
- R = radius of the ring [*m*]

Electric Field due to an infinite sheet: [*N/C*]

$$E = \frac{s}{2\in_0}$$

- E = electric field [*N/C*]
- s = charge per unit area [*C/m*^{2}}
- $\in_0 = 8.85 \times 10^{-12}$ [$C^2/N{\cdot}m^2$]

Electric Field inside a spherical shell: [*N/C*]

$$E = \frac{kqr}{R^3}$$

- E = electric field [*N/C*]
- q = charge [C]
- r = distance from center of sphere to the charge [*m*]
- R = radius of the sphere [*m*]

Electric Field outside a spherical shell: [*N/C*]

$$E = \frac{kq}{r^2}$$

- E = electric field [*N/C*]
- q = charge [C]
- r = distance from center of sphere to the charge [*m*]

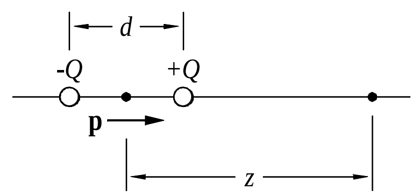
Average Power per unit area of an electric or magnetic field:

$$W/m^2 = \frac{E_m{}^2}{2\mu_0 c} = \frac{B_m{}^2 c}{2\mu_0}$$

- W = watts
- E_m = max. electric field [*N/C*]
- $m_0 = 4\pi \times 10^{-7}$
- $c = 2.99792 \times 10^8$ [*m/s*]
- B_m = max. magnetic field [*T*]

A positive charge moving in the same direction as the electric field direction loses potential energy since the potential of the electric field diminishes in this direction.
Equipotential lines cross EF lines at right angles.

Electric Dipole: Two charges of equal magnitude and opposite polarity separated by a distance d.



$$E = \frac{2k\mathbf{p}}{z^3}$$

$$E = \frac{1}{2p \in_0}\frac{\mathbf{p}}{z^3}$$

when $z \gg d$

- E = electric field [*N/C*]
- $k = 8.99 \times 10^9$ [$N{\cdot}m^2/C^2$]
- $\in_0$ = permittivity of free space 8.85×10^{-12} $C^2/N{\cdot}m^2$
- $\mathbf{p} = qd$ [$C{\cdot}m$] "electric dipole moment" in the direction negative to positive
- z = distance [m] from the dipole center to the point along the dipole axis where the electric field is to be measured

Deflection of a Particle in an Electric Field:

$$2ymv^2 = qEL^2$$

- y = deflection [*m*]
- m = mass of the particle [*kg*]
- d = plate separation [*m*]
- v = speed [*m/s*]
- q = charge [*C*]
- E = electric field [*N/C* or *V/m*
- L = length of plates [*m*]

Potential Difference between two Points: [volts *V*]

$$\Delta V = V_B - V_A = \frac{\Delta PE}{q} = -Ed$$

ΔPE = work to move a charge from *A* to *B* [N·m or J]
q = charge [C]
V_B = potential at *B* [V]
V_A = potential at *A* [V]
E = electric field [N/C or V/m
d = plate separation [m]

Electric Potential due to a Point Charge: [volts *V*]

$$V = k\frac{q}{r}$$

V = potential [volts V]
$k = 8.99 \times 10^9$ [N·m^2/C^2]
q = charge [C]
r = distance [m]

Potential Energy of a Pair of Charges: [*J*, *N·m* or *C·V*]

$$PE = q_2V_1 = k\frac{q_1q_2}{r}$$

V_1 is the electric potential due to q_1 at a point *P*
q_2V_1 is the work required to bring q_2 from infinity to point *P*

Work and Potential:

$$\Delta U = U_f - U_i = -W$$

$$U = -W_\infty$$

$$W = \mathbf{F}\cdot\mathbf{d} = Fd\cos q$$

$$W = q\int_i^f \mathbf{E}\cdot d\mathbf{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

$$V = -\int_i^f \mathbf{E}\cdot d\mathbf{s}$$

U = electric potential energy [*J*]
W = work done on a particle by a field [*J*]
$W_¥$ = work done on a particle brought from infinity (zero potential) to its present location [*J*]
$\mathbf{F}$ = is the force vector [*N*]
$\mathbf{d}$ = is the distance vector over which the force is applied[*m*]
F = is the force scalar [*N*]
d = is the distance scalar [*m*]
q = is the angle between the force and distance vectors
$d\mathbf{s}$ = differential displacement of the charge [*m*]
V = volts [*V*]
q = charge [*C*]

Flux: the rate of flow (of an electric field) [*N·m²/C*]

$$\Phi = \oint \mathbf{E}\cdot d\mathbf{A}$$

$$= \int E(\cos q)dA$$

Φ is the rate of flow of an electric field [*N·m²/C*]
$\oint$ integral over a closed surface
$\mathbf{E}$ is the electric field vector [*N/C*]
$\mathbf{A}$ is the area vector [*m²*] pointing outward normal to the surface.

Gauss' Law:

$$\in_0 \Phi = q_{enc}$$

$$\in_0 \oint \mathbf{E}\cdot d\mathbf{A} = q_{enc}$$

$\in_0 = 8.85 \times 10^{-12}$ [C^2/N·m^2]
Φ is the rate of flow of an electric field [*N·m²/C*]
q_{enc} = charge within the gaussian surface [*C*]
$\oint$ integral over a closed surface
$\mathbf{E}$ is the electric field vector [J]
$\mathbf{A}$ is the area vector [m^2] pointing outward normal to the surface.

CAPACITANCE

Parallel-Plate Capacitor:

$$C = k \in_0 \frac{A}{d}$$

C = capacitance [farads *F*]
κ = the dielectric constant (1)
$\in_0$ = permittivity of free space 8.85×10^{-12} $C^2/N\cdot m^2$
A = area of one plate [m^2]
d = separation between plates [*m*]

Cylindrical Capacitor:

$$C = 2pk \in_0 \frac{L}{\ln(b/a)}$$

C = capacitance [farads *F*]
κ = dielectric constant (1)
$\in_0 = 8.85 \times 10^{-12}$ $C^2/N\cdot m^2$
L = length [*m*]
b = radius of the outer conductor [*m*]
a = radius of the inner conductor [*m*]

Spherical Capacitor:

$$C = 4pk \in_0 \frac{ab}{b-a}$$

C = capacitance [farads *F*]
κ = dielectric constant (1)
$\in_0 = 8.85 \times 10^{-12}$ $C^2/N\cdot m^2$
b = radius, outer conductor [*m*]
a = radius, inner conductor [*m*]

Maximum Charge on a Capacitor: [Coulombs *C*]

$$Q = VC$$

Q = Coulombs [*C*]
V = volts [*V*]
C = capacitance in farads [*F*]

For capacitors connected in series, the charge *Q* is equal for each capacitor as well as for the total equivalent. If the dielectric constant κ is changed, the capacitance is multiplied by κ, the voltage is divided by κ, and *Q* is unchanged. In a vacuum $\kappa = 1$, When dielectrics are used, replace $\in_0$ with $\kappa \in_0$.

Electrical Energy Stored in a Capacitor: [Joules *J*]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

U = Potential Energy [*J*]
Q = Coulombs [*C*]
V = volts [*V*]
C = capacitance in farads [*F*]

Charge per unit Area: [*C/m*^{2}]

$$s = \frac{q}{A}$$

s = charge per unit area [*C/m*^{2}]
q = charge [*C*]
A = area [*m*^{2}]

Energy Density: (in a vacuum) [*J/m*^{3}]

$$u = \tfrac{1}{2} \epsilon_0 E^2$$

u = energy per unit volume [*J/m*^{3}]
ϵ_0 = permittivity of free space
8.85 × 10^{-12} $C^2/N \cdot m^2$
E = energy [*J*]

Capacitors in Series:

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$$

Capacitors in Parallel:

$$C_{eff} = C_1 + C_2 \ldots$$

Capacitors connected in series all have the same charge q. For parallel capacitors the total q is equal to the sum of the charge on each capacitor.

Time Constant: [seconds]

$$\tau = RC$$

t = time it takes the capacitor to reach 63.2% of its maximum charge [seconds]
R = series resistance [ohms Ω]
C = capacitance [farads *F*]

Charge or Voltage after *t* Seconds: [coulombs *C*]

charging:

$$q = Q\left(1 - e^{-t/t}\right)$$

$$V = V_S\left(1 - e^{-t/t}\right)$$

discharging:

$$q = Qe^{-t/t}$$

$$V = V_S \, e^{-t/t}$$

q = charge after t seconds [coulombs *C*]
Q = maximum charge [coulombs *C*] $Q = CV$
e = natural log
t = time [seconds]
t = time constant RC [seconds]
V = volts [*V*]
V_S = supply volts [*V*]

[Natural Log: when $e^b = x$, $\ln x = b$]

Drift Speed:

$$I = \frac{\Delta Q}{\Delta t} = \left(nqv_d A\right)$$

ΔQ = # of carriers × charge/carrier
Δt = time in seconds
n = # of carriers
q = charge on each carrier
v_d = drift speed in meters/second
A = cross-sectional area in meters^2

RESISTANCE

Emf: A voltage source which can provide continuous current [volts]

$$\varepsilon = IR + Ir$$

ε = emf open-circuit voltage of the battery
I = current [amps]
R = load resistance [ohms]
r = internal battery resistance [ohms]

Resistivity: [Ohm Meters]

$$r = \frac{E}{J}$$

$$r = \frac{RA}{L}$$

r = resistivity [$W \cdot m$]
E = electric field [*N/C*]
J = current density [*A/m*^{2}]
R = resistance [W ohms]
A = area [*m*^{2}]
L = length of conductor [*m*]

Variation of Resistance with Temperature:

$$r - r_0 = r_0 a (T - T_0)$$

r = resistivity [$W \cdot m$]
r_0 = reference resistivity [$W \cdot m$]
a = temperature coefficient of resistivity [K^{-1}]
T_0 = reference temperature
$T - T_0$ = temperature difference [*K* or °*C*]

CURRENT

Current Density: [*A/m*^{2}]

$$i = \int \mathbf{J} \cdot d\mathbf{A}$$

if current is uniform and parallel to $d\mathbf{A}$, then: $i = JA$

$$J = (ne)V_d$$

i = current [*A*]
J = current density [*A/m*^{2}]
A = area [*m*^{2}]
L = length of conductor [*m*]
e = charge per carrier
ne = carrier charge density [*C/m*^{3}]
V_d = drift speed [*m/s*]

Rate of Change of Chemical Energy in a Battery:

$$P = ie$$

P = power [*W*]
i = current [*A*]
e = emf potential [*V*]

Kirchhoff's Rules

1. The sum of the currents entering a junctions is equal to the sum of the currents leaving the junction.
2. The sum of the potential differences across all the elements around a closed loop must be zero.

Evaluating Circuits Using Kirchhoff's Rules

1. Assign current variables and direction of flow to all branches of the circuit. If your choice of direction is incorrect, the result will be a negative number. Derive equation(s) for these currents based on the rule that currents entering a junction equal currents exiting the junction.
2. Apply Kirchhoff's loop rule in creating equations for different current paths in the circuit. For a current path beginning and ending at the same point, the sum of voltage drops/gains is zero. When evaluating a loop in the direction of current flow, resistances will cause drops (negatives); voltage sources will cause rises (positives) provided they are crossed negative to positive—otherwise they will be drops as well.
3. The number of equations should equal the number of variables. Solve the equations simultaneously.

MAGNETISM

André-Marie **Ampére** is credited with the discovery of electromagnetism, the relationship between electric currents and magnetic fields.

Heinrich **Hertz** was the first to generate and detect electromagnetic waves in the laboratory.

Magnetic Force acting on a charge *q*: [Newtons *N*]

$$F = qvB\sin\theta$$

$$F = q\mathbf{v}\times\mathbf{B}$$

F = force [*N*]
q = charge [*C*]
v = velocity [*m/s*]
B = magnetic field [*T*]
q = angle between *v* and *B*

Right-Hand Rule: Fingers represent the direction of the magnetic force *B*, thumb represents the direction of *v* (at any angle to *B*), and the force *F* on a **positive** charge emanates from the palm. The direction of a magnetic field is from **north to south**. Use the *left* hand for a *negative* charge.

Also, if a **wire** is grasped in the right hand with the thumb in the direction of current flow, the fingers will curl in the direction of the magnetic field.

In a **solenoid** with current flowing in the direction of curled fingers, the magnetic field is in the direction of the thumb.

When applied to electrical flow caused by a **changing magnetic field**, things get more complicated. Consider the north pole of a magnet moving toward a loop of wire (magnetic field increasing). The thumb represents the north pole of the magnet, the fingers *suggest* current flow in the loop. However, electrical activity will serve to balance the change in the magnetic field, so that current will actually flow in the opposite direction. If the magnet was being withdrawn, then the *suggested* current flow would be decreasing so that the actual current flow would be in the direction of the fingers in this case to oppose the *decrease*. Now consider a cylindrical area of magnetic field going *into* a page. With the thumb pointing into the page, this would *suggest* an electric field orbiting in a clockwise direction. If the magnetic field was increasing, the actual electric field would be CCW in opposition to the increase. An electron in the field would travel opposite the field direction (CW) and would experience a negative change in potential.

Force on a Wire in a Magnetic Field: [Newtons *N*]

$$F = BI\ell\sin q$$

$$F = I\ell\times B$$

F = force [*N*]
B = magnetic field [*T*]
I = amperage [*A*]
ℓ = length [*m*]
q = angle between *B* and the direction of the current

Torque on a Rectangular Loop: [Newton·meters *N·m*]

$$\tau = NBIA\sin\theta$$

N = number of turns
B = magnetic field [*T*]
I = amperage [*A*]
A = area [m^2]
q = angle between B and the plane of the loop

Charged Particle in a Magnetic Field:

$$r = \frac{mv}{qB}$$

r = radius of rotational path
m = mass [*kg*]
v = velocity [*m/s*]
q = charge [*C*]
B = magnetic field [*T*]

Magnetic Field Around a Wire: [T]

$$B = \frac{\mu_0 I}{2\pi r}$$

B = magnetic field [*T*]
m_0 = the permeability of free space $4\pi\times10^{-7}$ T·m/A
I = current [*A*]
r = distance from the center of the conductor

Magnetic Field at the center of an Arc: [T]

$$B = \frac{m_0 i f}{4pr}$$

B = magnetic field [*T*]
m_0 = the permeability of free space $4\pi\times10^{-7}$ T·m/A
i = current [*A*]
f = the arc in radians
r = distance from the center of the conductor

Hall Effect: Voltage across the width of a conducting ribbon due to a Magnetic Field:

$$(ne)V_w h = Bi$$

$$v_d Bw = V_w$$

ne = carrier charge density [C/m^3]
V_w = voltage across the width [*V*]
h = thickness of the conductor [*m*]
B = magnetic field [*T*]
i = current [*A*]
v_d = drift velocity [*m/s*]
w = width [*m*]

Force Between Two Conductors: The force is attractive if the currents are in the same direction.

$$\frac{F_1}{\ell} = \frac{m_0 I_1 I_2}{2pd}$$

F = force [*N*]
ℓ = length [*m*]
m_0 = the permeability of free space $4\pi\times10^{-7}$ T·m/A
I = current [*A*]
d = distance center to center [*m*]

Magnetic Field Inside of a Solenoid: [Teslas *T*]

$$B = \mu_0 nI$$

B = magnetic field [*T*]
m_0 = the permeability of free space $4\pi\times10^{-7}$ T·m/A
n = number of turns of wire per unit length [*#/m*]
I = current [*A*]

Magnetic Dipole Moment: [*J/T*]

$$m = NiA$$

m = the magnetic dipole moment [*J/T*]
N = number of turns of wire
i = current [*A*]
A = area [m^2]

Magnetic Flux through a closed loop: [$T\cdot M^2$ or Webers]

$$\Phi = BA\cos q$$

B = magnetic field [*T*]
A = area of loop [m^2]
q = angle between B and the perpen-dicular to the plane of the loop

Magnetic Flux for a changing magnetic field: [$T \cdot M^2$ or Webers]

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}$$

B = magnetic field [T]
A = area of loop [m^2]

A Cylindrical Changing Magnetic Field

$$\oint \mathbf{E} \cdot d\mathbf{s} = E2p\,r = \frac{d\Phi_B}{dt}$$

$$\Phi_B = BA = Bp\,r^2$$

$$\frac{d\Phi}{dt} = A\frac{dB}{dt}$$

$$e = -N\frac{d\Phi}{dt}$$

E = electric field [N/C]
r = radius [m]
t = time [s]
Φ = magnetic flux [$T \cdot m^2$ or Webers]
B = magnetic field [T]
A = area of magnetic field [m^2]
dB/dt = rate of change of the magnetic field [T/s]
e = potential [V]
N = number of orbits

Faraday's Law of Induction states that the instantaneous emf induced in a circuit equals the rate of change of magnetic flux through the circuit. Michael **Faraday** made fundamental discoveries in magnetism, electricity, and light.

$$e = -N\frac{\Delta\Phi}{\Delta t}$$

N = number of turns
Φ = magnetic flux [$T \cdot m^2$]
t = time [s]

Lenz's Law states that the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change in magnetic flux through a circuit

Motional emf is induced when a conducting bar moves through a perpendicular magnetic field.

$$e = B\ell v$$

B = magnetic field [T]
ℓ = length of the bar [m]
v = speed of the bar [m/s]

emf Induced in a Rotating Coil:

$$e = NABw \sin wt$$

N = number of turns
A = area of loop [m^2]
B = magnetic field [T]
w = angular velocity [rad/s]
t = time [s]

Self-Induced emf in a Coil due to changing current:

$$e = -L\frac{\Delta I}{\Delta t}$$

L = inductance [H]
I = current [A]
t = time [s]

Inductance per unit length near the center of a solenoid:

$$\frac{L}{\ell} = m_0 n^2 A$$

L = inductance [H]
ℓ = length of the solenoid [m]
m_0 = the permeability of free space $4\pi \times 10^{-7}$ T·m/A
n = number of turns of wire per unit length [$\#/m$]
A = area [m^2]

Amperes' Law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = m_0 i_{enc}$$

B = magnetic field [T]
m_0 = the permeability of free space $4\pi \times 10^{-7}$ T·m/A
i_{enc} = current encircled by the loop[A]

Joseph **Henry**, American physicist, made improvements to the electromagnet.

James Clerk **Maxwell** provided a theory showing the close relationship between electric and magnetic phenomena and predicted that electric and magnetic fields could move through space as waves.

J. J. **Thompson** is credited with the discovery of the electron in 1897.

INDUCTIVE & RCL CIRCUITS

Inductance of a Coil: [H]

$$L = \frac{N\Phi}{I}$$

N = number of turns
Φ = magnetic flux [$T \cdot m^2$]
I = current [A]

In an RL Circuit, after one time constant ($t = L/R$) the current in the circuit is 63.2% of its final value, ε/R.

RL Circuit:

current rise:

$$I = \frac{V}{R}\left(1 - e^{-t/t_L}\right)$$

current decay:

$$I = \frac{V}{R}e^{-t/t_L}$$

U_B = Potential Energy [J]
V = volts [V]
R = resistance [W]
e = natural log
t = time [seconds]
t_L = inductive time constant L/R [s]
I = current [A]

Magnetic Energy Stored in an Inductor:

$$U_B = \tfrac{1}{2}LI^2$$

U_B = Potential Energy [J]
L = inductance [H]
I = current [A]

Electrical Energy Stored in a Capacitor: [Joules J]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

U_E = Potential Energy [J]
Q = Coulombs [C]
V = volts [V]
C = capacitance in farads [F]

Resonant Frequency: : The frequency at which $X_L = X_C$. In a **series**-resonant circuit, the impedance is at its minimum and the current is at its maximum. For a **parallel**-resonant circuit, the opposite is true.

$$f_R = \frac{1}{2p\sqrt{LC}}$$

$$w = \frac{1}{\sqrt{LC}}$$

f_R = Resonant Frequency [Hz]
L = inductance [H]
C = capacitance in farads [F]
w = angular frequency [rad/s]

Voltage, series circuits: [V]

$$V_C = \frac{q}{C} \quad V_R = IR$$

$$\frac{V_X}{X} = \frac{V_R}{R} = I$$

$$V^2 = V_R{}^2 + V_X{}^2$$

V_C = voltage across capacitor [*V*]
q = charge on capacitor [*C*]
f_R = Resonant Frequency [*Hz*]
L = inductance [*H*]
C = capacitance in farads [*F*]
R = resistance [*W*]
I = current [*A*]
V = supply voltage [*V*]
V_X = voltage across reactance [*V*]
V_R = voltage across resistor [*V*]

Phase Angle of a series RL or RC circuit: [degrees]

$$\tan f = \frac{X}{R} = \frac{V_X}{V_R}$$

$$\cos f = \frac{V_R}{V} = \frac{R}{Z}$$

(*f* would be negative in a capacitive circuit)

f = Phase Angle [degrees]
X = reactance [*W*]
R = resistance [*W*]
V = supply voltage [*V*]
V_X = voltage across reactance [*V*]
V_R = voltage across resistor [*V*]
Z = impedance [*W*]

Impedance of a series RL or RC circuit: [*W*]

$$Z^2 = R^2 + X^2$$

$$E = I\,Z$$

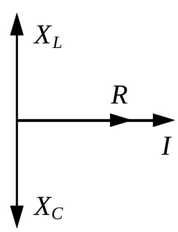
$$\frac{Z}{V} = \frac{X_C}{V_C} = \frac{R}{V_R}$$

$$Z = R \pm jX$$

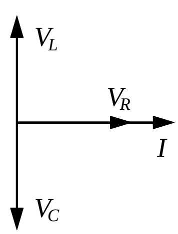
f = Phase Angle [degrees]
X = reactance [*W*]
R = resistance [*W*]
V = supply voltage [*V*]
V_X = voltage across reactance [*V*]
V_X = voltage across resistor [*V*]
Z = impedance [*W*]

Series RCL Circuits:

The Resultant Phasor $X = X_L - X_C$ is in the direction of the larger reactance and determines whether the circuit is inductive or capacitive. If X_L is larger than X_C, then the circuit is inductive and *X* is a vector in the upward direction.



In series circuits, the amperage is the reference (horizontal) vector. This is observed on the oscilloscope by looking at the voltage across the resistor. The two vector diagrams at right illustrate the phase relationship between voltage, resistance, reactance, and amperage.



Series RCL Impedance

$$Z^2 = R^2 + (X_L - X_C)^2 \qquad Z = \frac{R}{\cos f}$$

Impedance may be found by adding the components using vector algebra. By converting the result to polar notation, the phase angle is also found.

For multielement circuits, total each resistance and reactance before using the above formula.

Damped Oscillations in an RCL Series Circuit:

$$q = Qe^{-Rt/2L}\cos(w't + f)$$

where

$$w' = \sqrt{w^2 - (R/2L)^2}$$

$$w = 1/\sqrt{LC}$$

When R is small and ω′ ≈ ω:

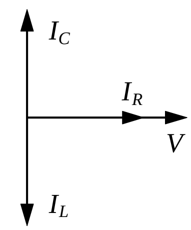
$$U = \frac{Q^2}{2C}e^{-Rt/L}$$

q = charge on capacitor [*C*]
Q = maximum charge [*C*]
e = natural log
R = resistance [*W*]
L = inductance [*H*]
w = angular frequency of the undamped oscillations [*rad/s*]
w = angular frequency of the damped oscillations [*rad/s*]
U = Potential Energy of the capacitor [*J*]
C = capacitance in farads [*F*]

Parallel RCL Circuits:

$$I_T = \sqrt{I_R{}^2 + (I_C - I_L)^2}$$

$$\tan f = \frac{I_C - I_L}{I_R}$$



To find total current and phase angle in multielement circuits, find *I* for each path and add vectorally. Note that when converting between current and resistance, a division will take place requiring the use of polar notation and resulting in a change of sign for the angle since it will be divided into (subtracted from) an angle of zero.

Equivalent Series Circuit: Given the *Z* in polar notation of a parallel circuit, the resistance and reactance of the equivalent series circuit is as follows:

$$R = Z_T \cos q \qquad X = Z_T \sin q$$

AC CIRCUITS

Instantaneous Voltage of a Sine Wave:

$$V = V_{\max}\sin 2\pi ft$$

V = voltage [*V*]
f = frequency [*Hz*]
t = time [s]

Maximum and rms Values:

$$I = \frac{I_m}{\sqrt{2}} \qquad V = \frac{V_m}{\sqrt{2}}$$

I = current [*A*]
V = voltage [*V*]

RLC Circuits:

$$V = \sqrt{V_R{}^2 + (V_L - V_C)^2}$$

$$\tan f = \frac{X_L - X_C}{R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

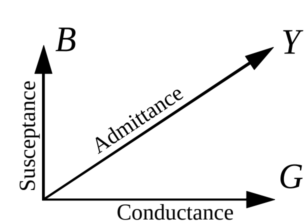
$$P_{avg} = IV\cos f$$

$$PF = \cos f$$

Conductance (G): The reciprocal of resistance in siemens (S).

Susceptance (B, B_L, B_C): The reciprocal of reactance in siemens (S).

Admittance (Y): The reciprocal of impedance in siemens (S).



ELECTROMAGNETICS

WAVELENGTH

$c = \lambda f$ $c = E / B$ $1Å = 10^{-10}$m	c = speed of light 2.998×10^{8} m/s λ = wavelength [m] f = frequency [Hz] E = electric field [N/C] B = magnetic field [T] Å = (angstrom) unit of wavelength equal to 10^{-10} m m = (meters)

WAVELENGTH SPECTRUM

BAND	METERS	ANGSTROMS
Longwave radio	1 - 100 km	10^{13} - 10^{15}
Standard Broadcast	100 - 1000 m	10^{12} - 10^{13}
Shortwave radio	10 - 100 m	10^{11} - 10^{12}
TV, FM	0.1 - 10 m	10^{9} - 10^{11}
Microwave	1 - 100 mm	10^{7} - 10^{9}
Infrared light	0.8 - 1000 µm	8000 - 10^{7}
Visible light	360 - 690 nm	3600 - 6900
violet	360 nm	3600
blue	430 nm	4300
green	490 nm	4900
yellow	560 nm	5600
orange	600 nm	6000
red	690 nm	6900
Ultraviolet light	10 - 390 nm	100 - 3900
X-rays	5 - 10,000 pm	0.05 - 100
Gamma rays	100 - 5000 fm	0.001 - 0.05
Cosmic rays	< 100 fm	< 0.001

Intensity of Electromagnetic Radiation [watts/m^2]:

$$I = \frac{P_s}{4p r^2}$$

I = intensity [w/m^2]
P_s = power of source [watts]
r = distance [m]
$4pr^2$ = surface area of sphere

Force and Radiation Pressure on an object:

a) if the light is totally absorbed:

$$F = \frac{IA}{c} \qquad P_r = \frac{I}{c}$$

b) if the light is totally reflected back along the path:

$$F = \frac{2IA}{c} \qquad P_r = \frac{2I}{c}$$

F = force [N]
I = intensity [w/m^2]
A = area [m^2]
P_r = radiation pressure [N/m^2]
c = 2.99792 × 10^8 [m/s]

Poynting Vector [watts/m^2]:

$$S = \frac{1}{m_0}EB = \frac{1}{m_0}E^2$$

$$cB = E$$

m_0 = the permeability of free space $4\pi \times 10^{-7}$ T·m/A
E = electric field [N/C or V/M]
B = magnetic field [T]
c = 2.99792 × 10^8 [m/s]

LIGHT

Indices of Refraction:

Quartz:	1.458
Glass, crown	1.52
Glass, flint	1.66
Water	1.333
Air	1.000 293

Angle of Incidence: The angle measured from the perpendicular to the face or from the perpendicular to the tangent to the face

Index of Refraction: Materials of greater density have a higher index of refraction.

$$n \equiv \frac{c}{v}$$

n = index of refraction
c = speed of light in a vacuum 3 × 10^8 m/s
v = speed of light in the material [m/s]

$$n = \frac{\lambda_0}{\lambda_n}$$

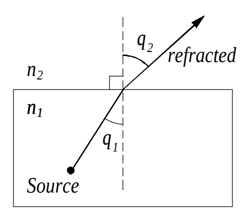
l_0 = wavelength of the light in a vacuum [m]
l_n = its wavelength in the material [m]

Law of Refraction: **Snell's Law**

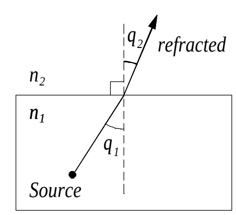
$$n_1 \sin q_1 = n_2 \sin q_2$$

n = index of refraction
q = angle of incidence

traveling to a region of lesser density: $q_2 > q_1$

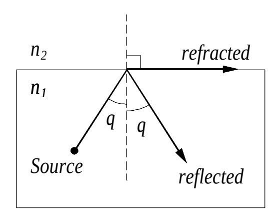


traveling to a region of greater density: $q_2 < q_1$



Critical Angle: The maximum angle of incidence for which light can move from n_1 to n_2

$$\sin\theta_c = \frac{n_2}{n_1} \quad \text{for } n_1 > n_2$$



Sign Conventions: When M is negative, the image is inverted. p is positive when the object is in front of the mirror, surface, or lens. Q is positive when the image is in front of the mirror or in back of the surface or lens. f and r are positive if the center of curvature is in front of the mirror or in back of the surface or lens.

Magnification by spherical mirror or thin lens. A negative m means that the image is inverted.

$$M = \frac{h'}{h} = -\frac{i}{p}$$

h' = image height [m]
h = object height [m]
i = image distance [m]
p = object distance [m]

Plane Refracting Surface:

plane refracting surface:

$$\frac{n_1}{p} = -\frac{n_2}{i}$$

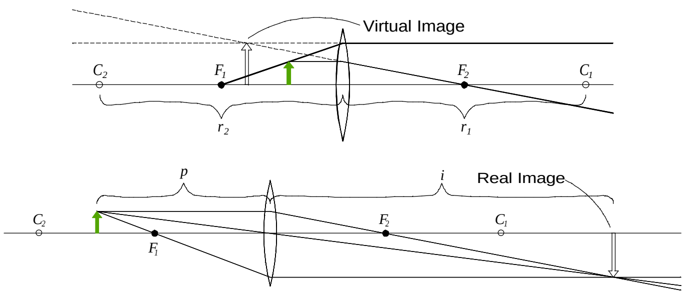
p = object distance
i = image distance [*m*]
n = index of refraction

Lensmaker's Equation for a thin lens in air:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

f = focal length [*m*]
i = image distance [*m*]
p = object distance [*m*]
n = index of refraction

r_1 = radius of surface nearest the object[*m*]
r_2 = radius of surface nearest the image [*m*]



Thin Lens when the thickest part is thin compared to *p*.

i is negative on the left, positive on the right

$$f = \frac{r}{2}$$

f = focal length [*m*]
r = radius [*m*]

Converging Lens

f is positive (left)
r_1 and r_2 are positive in this example

Diverging Lens

f is negative (right)
r_1 and r_2 are negative in this example

Two-Lens System

Perform the calculation in steps. Calculate the image produced by the first lens, ignoring the presence of the second. Then use the image position relative to the second lens as the object for the second calculation ignoring the first lens.

Spherical Refracting Surface

This refers to two materials with a single refracting surface.

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

$$M = \frac{h'}{h} = -\frac{n_1 i}{n_2 p}$$

p = object distance
i = image distance [*m*] (positive for real images)
f = focal point [*m*]
n = index of refraction
r = radius [*m*] (positive when facing a convex surface, unlike with mirrors)
M = magnification
h' = image height [*m*]
h = object height [*m*]

Constructive and Destructive Interference by Single and Double Slit Defraction and Circular Aperture

Young's double-slit experiment (bright fringes/dark fringes):

Double Slit

Constructive:

$$\Delta L = d \sin q = m\, l$$

Destructive:

$$\Delta L = d \sin q = (m + \tfrac{1}{2})\, l$$

d = distance between the slits [*m*]
q = the angle between a normal line extending from midway between the slits and a line extending from the midway point to the point of ray intersection.
m = fringe order number [*integer*]
l = wavelength of the light [*m*]
a = width of the single-slit [*m*]
DL = the difference between the distance traveled of the two rays [*m*]
I = intensity @ *q* [*W/m*^{2}]
I_m = intensity @ *q* = 0 [*W/m*^{2}]
d = distance between the slits [*m*]

Intensity:

$$I = I_m (\cos^2 b)\left(\frac{\sin a}{a}\right)^2$$

$$b = \frac{p\, d}{l} \sin q$$

$$a = \frac{p\, a}{l} \sin q$$

Single-Slit

Destructive:

$$a \sin q = m\, l$$

Circular Aperture

1st Minimum:

$$\sin q = 1.22 \frac{g}{dia.}$$

In a circular aperture, the 1st minimum is the point at which an image can no longer be resolved.

A **reflected ray** undergoes a **phase shift** of 180° when the reflecting material has a greater index of refraction *n* than the ambient medium. Relative to the same ray without phase shift, this constitutes a path difference of λ/2.

Interference between Reflected and Refracted rays

from a thin material **surrounded** by another medium:

Constructive:

$$2nt = (m + \tfrac{1}{2})\lambda$$

Destructive:

$$2nt = m\lambda$$

n = index of refraction
t = thickness of the material [*m*]
m = fringe order number [*integer*]
l = wavelength of the light [*m*]

If the thin material is between two different media, one with a higher *n* and the other lower, then the above constructive and destructive formulas are reversed.

Wavelength within a medium:

$$\lambda_n = \frac{\lambda}{n}$$

$$c = n\, l_n\, f$$

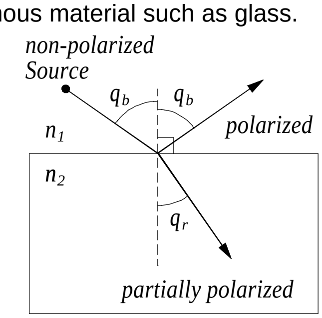
l = wavelength in free space [*m*]
l_n = wavelength in the medium [*m*]
n = index of refraction
c = the speed of light 3.00 × 10^8 [*m/s*]
f = frequency [*Hz*]

Polarizing Angle:

by **Brewster's Law**, the angle of incidence that produces complete polarization in the reflected light from an amorphous material such as glass.

$$\tan q_B = \frac{n_2}{n_1}$$

$$q_r + q_B = 90°$$



n = index of refraction
q_B = angle of incidence producing a 90° angle between reflected and refracted rays.
q_r = angle of incidence of the refracted ray.

Intensity of light passing through a polarizing lense:

[*Watts/m*^{2}]

initially unpolarized: $I = \frac{1}{2} I_0$

initially polarized:

$$I = I_0 \cos^2 q$$

I = intensity [*W/m*^{2}]
I_0 = intensity of source [*W/m*^{2}]
q = angle between the polarity of the source and the lens.